THE MATRIX CONDITION NUMBER IN THE NEUTRON NOISE DIAGNOSTICS

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Abstract

It is recognized, that the matrix condition number plays important role in the error of estimation of core barrel motion. Matrix condition numbers were estimated for the core barrel movement measured in Paks NPP, Kalinin NPP and Borssele NPP. Results show, that spectral decomposition leads to an increases the statistical error. It is suggested to eliminate one of the freedom of the system, before making spectral decomposition.

1 SPECTRAL DECOMPOSITION

The core barrel motion can be detected and evaluated from the fluctuation measured by ex–core neutron detectors arranged around the reactor vessel of a NPP (see [8], [11], [12], [3]). The basis of the method is very simple. Due to any layer between the core and biological shielding with thickness of Δx and attenuation coefficient μ an attenuation of neutron fluxes occurs (see [4], [1]).

Figure 1. shows the typical arrangement of ex–core neutron detectors in a PWR. It is clear that one of the possible force for a pendular motion is the jet of coolant from the loops. The pendular movement causes a variation $\delta x(t)$ of gap Δx. It is also well known, that there is always some reactivity perturbation in the core. On the basis of these two noise sources a method of spectral decomposition has been elaborated by Dragt and Turkcan [4].

In that model the signal measured by neutron detector consists of two parts:

- reactivity fluctuation
- pendular core barrel motion.

The direction of the motion can differ from the direction of the given neutron detector. In this case the given detector will sense only a projection of the motion.

According to the original idea of spectral decomposition method [4] the signal of any ex–core detector can be written as:

$$i(t) = \delta r(t) + \mu\, \delta y(t) \cos(\Theta_i - \Theta^{'}) + w(t) \tag{1}$$

$\delta r(t)$ – is the reactivity fluctuation independent from CBM
$\delta y(t)$ – is the amplitude of CBM
Θ_i – is the direction of the given detector
$\Theta^{'}$ – is the direction of CBM
$w(t)$ – white noise from electronics

From here Dragt and Türkcan [4] got the cross power spectral density between any pairs of ex–core neutron detectors.

$$\begin{aligned}(S_{ij})_{Re} = A_1 + A_2 \cos(\Theta_i + \Theta_j) + A_3 \sin(\Theta_i + \Theta_j) \\ + A_4 \cos(\Theta_i - \Theta_j) + A_5 (\cos\Theta_i + \cos\Theta_j) \\ + A_6 (\sin\Theta_i + \sin\Theta_j)\end{aligned} \tag{2}$$

A_1 – is the reactivity part which includes also the spectrum

A_j contains partly the core barrel motion characteristics, partly their cross terms with reactivity if such exist. (details can be found in [4].) We are not interested in their explicit form for further derivations, it does not affect on our estimation.

We rewrite (2) in a matrix form:

$$\vec{S} = D\,\vec{A} \tag{3}$$

$\vec{S}$ – input vector, the measured quantities (CPSD)
D – detector–matrix
$\vec{A}$ – output vector estimated by SPEC–DEC

2 THE LEAST SQUARES METHOD

To calculate all six components of vector $\vec{A}$, we need at least four detectors, because from this number we can get six different pairs of detectors.

If there are more than four detectors, we can obtain the best fit using the method of least squares, thereto we minimize the next quantity:

$$Q = \sum_{l=1}^{N} (S_l - \sum_{m=1}^{6} D_{l,m} A_m)^2 \tag{4}$$

After the minimalization we obtain the normal equations:

$$(D^T D)\vec{A} = D^T \vec{S} \tag{5}$$

For the following derivation we introduce some notions, which we borrowed from [5].

The definition of the norm of vector $\vec{A}$ is the following quantity:

$$||\vec{A}|| = \max_i \{|A_i|, i = 1, 2, ...n\} \tag{6}$$

The norm of matrix D is the following quantity:

$$||D|| = \max_i \{\sum_j |D_{ij}|, i = 1, 2, ...n\} \tag{7}$$

From (6) and from (7) it follows, that:

$$||D|| \cdot ||\vec{A}|| \geq ||D\vec{A}||, \tag{8}$$

since:

$$\begin{aligned} ||D\vec{A}|| = \max_i \{\textstyle\sum_j |D_{ij}A_j|\} = \max_i \{\textstyle\sum_j |D_{ij}||A_j|\} \\ \leq \max_j \{|A_j|\} \cdot \max_i \{\textstyle\sum_j |D_{ij}|\} = ||D|| \cdot ||\vec{A}|| \end{aligned} \tag{9}$$

Consider, what is the consequence if any component of the vector $\vec{S}$ varies. The normal equations for varied vectors:

$$(D^T D)\vec{A}_\sim = D^T \vec{S}_\sim \tag{10}$$

Let us calculate the difference:

$$\vec{A}_\sim - \vec{A} = (D^T D)^{-1} (D^T \vec{S}_\sim - D^T \vec{S}) \tag{11}$$

Its norm is:

$$||\vec{A}_\sim - \vec{A}|| \leq ||(D^T D)^{-1}|| \cdot ||D^T \vec{S}_\sim - D^T \vec{S}|| \tag{12}$$

For the norm of vector $D^T \vec{S}$ we obtain:

$$||D^T D|| \cdot ||\vec{A}|| \geq ||D^T \vec{S}|| \tag{13}$$

From (12) and (13) we get:

$$||\vec{A}_\sim - \vec{A}|| \cdot ||D^T \vec{S}|| \leq ||(D^T D)^{-1}|| \cdot ||D^T D|| \cdot ||\vec{A}|| \cdot ||D^T \vec{S}_\sim - D^T \vec{S}|| \tag{14}$$

And:

$$\frac{||\vec{A}_\sim - \vec{A}||}{||\vec{A}||} \leq k_{D^T D} \cdot \frac{||D^T \vec{S}_\sim - D^T \vec{S}||}{||D^T \vec{S}||} \tag{15}$$

where:

$$k_{D^T D} = ||(D^T D)^{-1}|| \cdot ||D^T D|| \tag{16}$$

is the **condition number** of matrix $D^T D$.
The definitions of norms involve, that:

$$\frac{|(\vec{A}_\sim - \vec{A})_i|}{|\vec{A}_i|} \leq k_{D^T D} \cdot \left[\frac{|(D^T \vec{S}_\sim - D^T \vec{S})_j|}{|(D^T \vec{S})_j|} \right] \tag{17}$$

Formula (17) contains the result of the derivation. Its meaning is that for a system defined by (3), where $\vec{S}$ is the input data while $\vec{A}$ is the output, the ratio of relative errors of output to input is limited by the condition number and can be equal to condition number of matrix $D^T D$. From here we do conclude: when

considering all possible variation of the input, the statistical error of the estimate $\vec{A}$ has a measure defined by (17). More details one can find in [5].

3 APPLYING ERROR ESTIMATION TO REAL CASES

Hungarian researchers carried out core barrel measurements in Paks NPP, Kalinin NPP and recently in Borssele NPP (see [1], [6], [9]). A series of publications dealt with CBM monitoring in WWER type reactor ([1], [10], [9], [6]).

Let us consider the condition number for those measurements.

3.1 Condition number estimation for measurements in Paks NPP

In WWER-440 reactors ex–core detectors of the safety system are positioned as shown on Figure 3. (These detectors are used for noise diagnostics as well.)

Applying the derivation from the previous section, using the detectors positions of WWER – 440 type reactors as:

$$\Theta_1 := \frac{1}{6}\pi,\ \Theta_2 := \frac{1}{4}\pi,\ \Theta_3 := \frac{5}{6}\pi,$$
$$\Theta_4 := \frac{11}{12}\pi,\ \Theta_5 := \frac{3}{2}\pi,\ \Theta_6 := \frac{19}{12}\pi,$$

we get matrix $D^T D$:

$$\begin{bmatrix} 15 & -.00014 & -.00008 & -3.0000 & .00016 & -.0001 \\ -.00014 & 7.5003 & .00019 & -.00007 & 1.0607 & -2.5606 \\ -.00008 & .00019 & 7.5003 & .00006 & -2.5607 & -1.0606 \\ -3.0000 & -.00007 & .00006 & 6.0003 & -.00003 & 0 \\ .00016 & 1.0607 & -2.5607 & -.00003 & 12.001 & -.00015 \\ -.0001 & -2.5606 & -1.0606 & 0 & -.00015 & 12.000 \end{bmatrix}$$

Now we can calculate the condition number for WWER – 440:

$$k_{D^T D} = 4.0$$

3.2 Condition number estimation for WWER – 1000 type reactors

Detector positions are slightly different :

$$\Theta_1 := 0\,,\ \Theta_2 := \frac{1}{15}\pi,\ \Theta_3 := \frac{2}{3}\pi,$$
$$\Theta_4 := \frac{11}{15}\pi,\ \Theta_5 := \frac{4}{3}\pi,\ \Theta_6 := \frac{7}{5}\pi,$$

we get matrix $D^T D$:

$$\begin{bmatrix} 15 & .00008 & .00008 & -3.0000 & .00017 & .0003 \\ .00008 & 7.5000 & .00001 & .00002 & -2.7135 & -0.8816 \\ .00008 & .00001 & 7.5000 & -.0001 & -0.8816 & 2.7135 \\ -3.0000 & .00002 & -.0001 & 6.0000 & .00008 & 0 \\ .00017 & -2.7135 & -0.8816 & .00008 & 12.001 & .0002 \\ .0003 & -0.8816 & 2.7135 & 0 & .0002 & 12.000 \end{bmatrix}$$

Now we can calculate the condition number for WWER – 1000:

$$k_{D^TD} = 4.0$$

3.3 Condition number estimation for measurements in Borssele

Let us apply the above derivation to the original work [4], i.e. for Borssele NPP CBM estimation. From the geometry of detector position in that reactor:

$$\Theta_1 := \frac{5}{18}\pi,\ \Theta_2 := \frac{3}{5}\pi,\ \Theta_3 := \frac{7}{9}\pi,$$

$$\Theta_4 := \frac{23}{18}\pi,\ \Theta_5 := \frac{8}{5}\pi,\ \Theta_6 := \frac{16}{9}\pi$$

The D^TD matrix reads as:

$$\begin{bmatrix} 15 & .80897 & .58783 & -3.0002 & 0 & 0 \\ .80897 & 8.5946 & .81775 & -4.0451 & -.00002 & -.00001 \\ .58783 & .81775 & 6.4056 & -2.9390 & .00009 & 0 \\ -3.0002 & -4.0451 & -2.9390 & 7.0000 & -.00004 & 0 \\ 0 & -.00002 & .00009 & -.00004 & 8.7638 & -2.3513 \\ 0 & -.00001 & 0 & 0 & -2.3513 & 15.236 \end{bmatrix}$$

Consequently the condition number for the detector arrangement in Borssele NPP is:

$$k_{D^TD} = 10.405$$

4 DISCUSSION

From these results it can be seen the condition number depends on detector position for SPEC–DEC method. One can see from the calculated condition numbers that application of SPEC–DEC method can lead to higher statistical error.

We believe that large condition numbers are a consequence of the similarity of the component of CPSD(S_j) coming from reactivity effect.

The solution might be, that we eliminate the reactivity term before the start of analysis. A possible way for that is subtracting the average of signals of in–core neutron detectors from all signals of ex–core neutron detectors.

This elimination of the reactivity term leads us to a model which contains only terms of pendular motion. For this new model of decomposition we obtained the condition number of least square fit is equal to 1,25 for the geometry of detectors in Paks [1].

[1]Calculations were made by the programmes **Maple V and MATLAB**.[2],[7]

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Figure 1: The core barrel motion

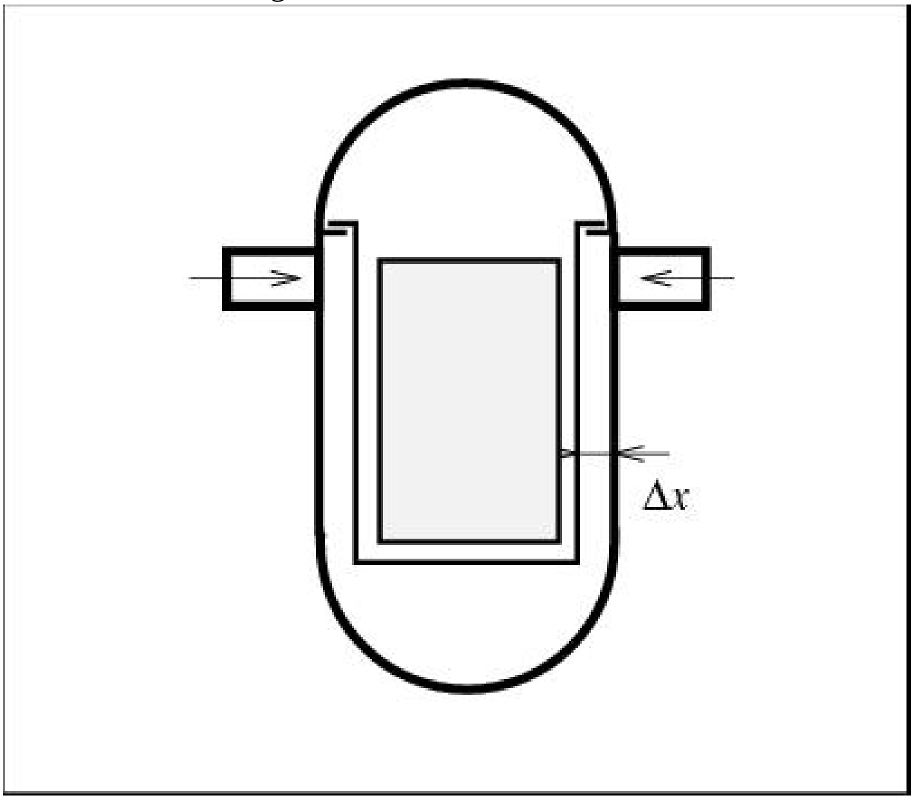

Figure 2: The projection of the motion

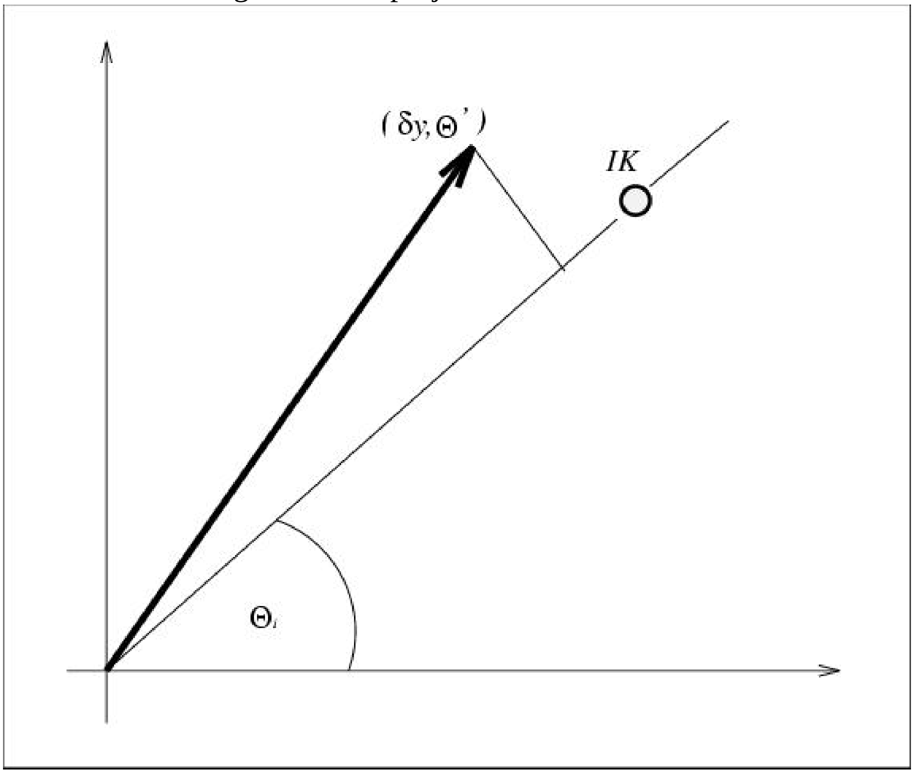

Figure 3: Geometry of detectors in Paks

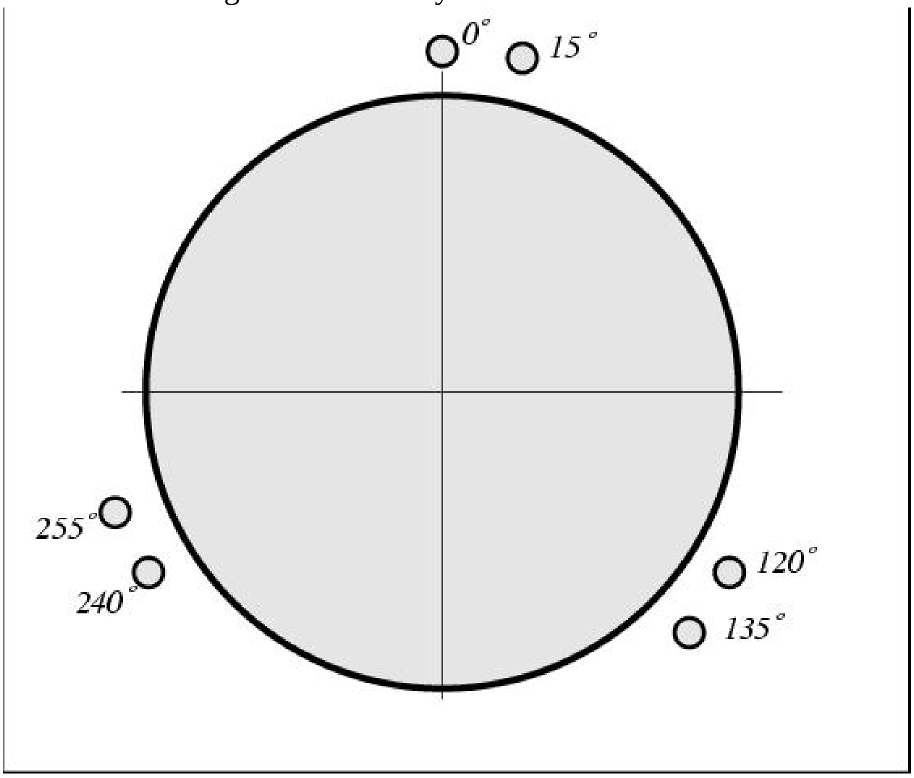

Table 1: Summary table

THE TABLE OF ESTIMATED CONDITION NUMBERS

Model	*Geometry*	k_{D^TD}
Motion+Reactivity	6 detectors , Paks	4,0
Motion+Reactivity	5 detectors , Paks	155,8
Motion+Reactivity	4 detectors , Paks	327,0
Motion+Reactivity	4 detectors , Paks	19 723
Motion+Reactivity	6 detectors , Kalinin	4,0
Motion+Reactivity	5 detectors , Kalinin	267,0
Motion+Reactivity	4 detectors , Kalinin	411,4
Motion+Reactivity	4 detectors , Kalinin	63 179
Motion+Reactivity	4 detectors , 90^o	8,0
Motion+Reactivity	6 detectors , Borssele	10,4
Motion	6 detectors , Paks	1,25
Motion	5 detectors , Paks	3,39
Motion	4 detectors , Paks	4,5
Motion	4 detectors , Paks	13,69
Motion	6 detectors , Borssele	6,32
Motion	4 detectors , 90^o	2,0
Motion	3 detectors , 120^o	2,0