A TRACKING CONTROLLER FOR FLEXIBLE JOINT ROBOTS BASED ON SEPARATION PRINCIPLE *

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Abstract

This paper deals with the tracking control problem of flexible joint robots under the assumption that the link and robot positions are the only measured signals. A globally asymptotically stable state feedback controller and a nonlinear observer is proposed which guarantees the semiglobal asymptotic tracking of the system obtained by the designed dynamic output feedback controller. Examples illustrate the result.

Keywords : Flexible joints; nonlinear systems; nonlinear observers; exponential stability.

1 Introduction

The tracking control problem of robot manipulators having elastic joints has attacted considerable attention in the past decade (see e.g. [9], [8], [5], [6] and the references therein). The stabilizing controllers have been design under the assumption of different measurements: in [6] the link positions and speeds, in [5] only the link positions, while in [4] the link and motor positions are assumed to be available from measurement. All of these investigations make use of a nonlinear observer having (locally or semiglobally) asymptotically stable error dynamics. It is known that having such an observer does not guarantee the stability of the control loop when the observer is used in conjunction with a stabilizing feedback controller, i. e. the separation principle which is well-established for linear systems, is not valid in general for nonlinear systems. Accordingly, the controller is designed in the above mentioned pagers directly in terms of the estimated variables. Lately, several results have been published about the applicability of the separation principle for the stabilization of certain classes of nonlinear time-invariant systems about an equilibrium state [10], [2]. These results have motivated the construction of the dynamic output feedback controller of the present paper, based on the separation principle. The advantage of this approach is in the possibility of designing the controller parameters independently of the observer; the parameters of the observer have to be fitted accordingly.

The structure of the paper is as follows. In Section II we formulate the problem, in Section III a globally stabilizing state feedback is designed by backstepping, in Section IV a nonlinear observer is proposed. Section V contains the main result of the paper showing the validity of the separation principle. Section VI contains two illustrative examples, and the conclusion is drawn up in Section VII.

We use standard notation in the paper. In particular, $\parallel x \parallel$ denotes the Euclidean norm of the vector x, $\parallel A \parallel$ denotes the induced matrix norm of the matrix A. A^T denotes the transpose of the matrix A, $\lambda_m(A)$ and $\lambda_M(A)$ denote the minimum and maximum eigenvalues of symmetric matrix A, respectively.

2 Problem statement

In this paper we consider the following model of elastic joint robots (detailed description of dynamic models of such robots is reported in [9], [8].

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Assuming that the motion of the actuator rotors may be considered as pure rotations with respect to an inertial frame, the model of an elastic joint robot is given by

$$\begin{aligned} B_1(q_1)\ddot{q}_1 + C_1(q_1,\dot{q}_1)\dot{q}_1 + K(q_1-q_2) + h(q_1) &= 0 \\ B_3\ddot{q}_2 - K(q_1-q_2) &= u \end{aligned} \tag{1}$$

where

$$C_1(q_1,\dot{q}_1)\dot{q}_1 = \dot{B}_1(q_1)\dot{q}_1 - \frac{1}{2}\frac{\partial\, \dot{q}_1^T B_1(q_1)\dot{q}_1}{\partial q_1} \tag{2}$$

and $\dot{B}_1 - 2C_1$ is a skew-symmetric matrix, for a suitable definition of C_1 (see [5]). For rotational joints there exist positive constants B_{1m}, B_{1M} and C_{1M} such that

$$B_{1m} \le \lambda_m(B_1(q_1)) \le \|B_1(q_1)\| \le \lambda_M(B_1(q_1)) \le B_{1M}, \quad \forall q_1 \in R^n \tag{3}$$

$$\left\|C_1(q_1,\dot{q}_1)\right\| \le C_{1M}\left\|\dot{q}_1\right\|, \quad \forall q_1,\dot{q}_1 \in R^n. \tag{4}$$

Matrix C_1 has the property (see [5])

$$C_1(q_1,y)z = C_1(q_1,z)y \tag{5}$$

where y and z are $n\times 1$ vectors.

Assume that the variables q_1 and q_2 are available for measurement so that the output of the system is given by

$$y_1 = q_1, \quad y_2 = q_2. \tag{6}$$

Let $q_d : [0,\infty) \to R^n$ be a given countinuos and bounded function with continuous and bounded derivatives up to the order 4, which is referred to as the desired reference trajectory for the link position. The problem is to design an dynamic output feedback controller for which the corresponding solution of (1) satisfies the condition $\lim_{t\to\infty}(q_1(t)-q_d(t)) = 0$ and $\lim_{t\to\infty}(\dot{q}_1(t) - \dot{q}_d(t)) = 0$. This problem can be traced back to the stabilization problem of the equilibrium state coinciding with the origin in the following standard way: we introduce new variables z_i, $i = 1,...,4$ by the definition

$$z_1 = q_1 - q_d, \;\; z_2 = \dot{q}_1 - \dot{q}_d, \;\; z_3 = q_2 - p_d, \;\; z_4 = \dot{q}_2 - \dot{p}_d, \;\; v = u - u_d, \tag{7}$$

where p_d and u_d are time-dependent functions to be fixed later. In this variables model (1) becomes

$$\begin{aligned} \dot{z}_1 &= z_2 \\ B_1(z_1+q_d)\dot{z}_2 &= -C_1(z_1+q_d, z_2+\dot{q}_d)(z_2+\dot{q}_d) \\ &\quad - h_1(z_1+q_d) - K(z_1+q_d-(z_3+p_d)) - B_1(z_1+q_d)\ddot{q}_d \\ \dot{z}_3 &= z_4 \\ \dot{z}_4 &= B_3^{-1}K((z_1+q_d)-(z_3+p_d)) + B_3^{-1}v + B_3^{-1}u_d - \ddot{p}_d \end{aligned} \tag{8}$$

If p_d and u_d are chosen as

$$\begin{aligned} p_d &= K^{-1}(C_1(q_d,\dot{q}_d)\dot{q}_d + h_1(q_d) + Kq_d + B_1(q_d)\ddot{q}_d) \\ u_d &= B_3\ddot{p}_d - K(q_d - p_d) \end{aligned}$$

then $z_i = 0$, $i = 1,...,4$, $v = 0$ is an equlibrium solution of (8). (Note that both p_d and u_d are functions of the derivatives of q_d up to the order 4.)

3 Stabilization by state feedback

Model (8) can be considered as the cascade connection of four subsystems in which the output of the ith subsystem is taken to be z_i and it is considered as the "artificial" input of the previous system. Therefore, the (time-dependent) state-feedback can be constructed by a standard backsteping procedure (see e.g. [3], [10]). In what follows, let K_i, $i = 1,...,4$ denote symmetric positive definite matrices. Starting with the first subsystem of (8) we consider z_2 as an "artificial" control variable and define its desired value z_2^* as

$$z_2^* = -K_1 z_1.$$

If

$$e_1 = z_1, e_2 = z_2 - z_2^*,$$

then

$$\dot{e}_1 = -K_1 e_1 + e_2. \tag{9}$$

Now, in the second eqution of (8), z_3 is taken as the artificial input. If its desired value is given by

$$\begin{aligned} z_3^* &= K^{-1}(C_1(z_1+q_d, z_2+\dot{q}_d)(z_2^*+\dot{q}_d) + h_1(z_1+q_d) + K(z_1+q_d) + \\ &\quad + B_1(z_1+q_d)\ddot{q}_d - Kp_d - B_1(z_1+q_d)K_1 z_2 - K_2(z_2+K_1 z_1) - z_1) \\ &= : \Phi_1(z_1, z_2, q_d, \dot{q}_d, \ddot{q}_d) \end{aligned} \tag{10}$$

then, by introducing the notation $e_3 = z_3 - z_3^*$, we get for e_2 the following equation:

$$B_1(z_1+q_d)\dot{e}_2 = -C_1(z_1+q_d, z_2+\dot{q}_d)e_2 - K_2 e_2 - e_1 + Ke_3. \tag{11}$$

Using z_4 as an artificial input in the third equation of (8) and the definitions of e_3, z_3^*, we obtain that

$$\dot{e}_3 = z_4 - z_4^* + z_4^* - \frac{d}{dt}\Phi_1(z_1, z_2, q_d, \dot{q}_d, \ddot{q}_d).$$

Let $e_4 = z_4 - z_4^*$ and

$$\begin{aligned} z_4^* &= \frac{d}{dt}\Phi_1(z_1, z_2, q_d, \dot{q}_d, \ddot{q}_d) - K_3(z_3 - z_3^*) - K(z_2 - z_2^*) \\ &= : \Phi_2(z_1, z_2, z_3, q_d, \dot{q}_d, \ddot{q}_d, \dddot{q}_d). \end{aligned} \tag{12}$$

then

$$\dot{e}_3 = -K_3 e_3 + e_4 - Ke_2. \tag{13}$$

Moreover,

$$\dot{e}_4 = \dot{z}_4 - \dot{z}_4^* = B_3^{-1}K(z_1 - z_3) + B_3^{-1}v - \frac{d}{dt}\Phi_2(z_1, z_2, z_3, q_d, \dot{q}_d, \ddot{q}_d, \dddot{q}_d).$$

Define

$$\begin{aligned} v^* &= -K(z_1 - z_3) + B_3\frac{d}{dt}\Phi_2(z_1, z_2, z_3, q_d, \dot{q}_d, \ddot{q}_d, \dddot{q}_d) - B_3 K_4(z_4 - z_4^*) - B_3(z_3 - z_3^*) \\ &= : \Phi_3(z_1, z_2, z_3, z_4, q_d, ..., q_d^{(IV)}), \end{aligned} \tag{14}$$

then

$$\dot{e}_4 = -K_4 e_4 - e_3. \tag{15}$$

Remark 3.1. Observe, that Φ_3 is continuously differentiable in its first n (vector) variables and $\Phi_3(0,0,0,0,q_d,...,q_d^{(IV)}) = 0$ therefore it is Lipschitz continuous in these variables in any compact set $Q \subset R^{4n}$.

Theorem 1. System (9)-(11)-(13)-(15) is globally exponentially stable about the origin.

Proof. Let us introduce the notation $e^T = (e_1^T, e_2^T, e_3^T, e_4^T)$ and consider the candidate Lyapunov function

$$V_E(t,e) = \frac{1}{2}[e_1^T e_1 + e_2^T B_1(q_1(t))e_2 + e_3^T e_3 + e_4^T e_4].$$

Obviously

$$\min\{\frac{1}{2}, \frac{1}{2}B_{1m}\}\|e\|^2 \le V_E(t,e) \le \max\{\frac{1}{2}, \frac{1}{2}B_{1M}\}\|e\|^2.$$

Using property (5), a straightforward calculation shows that the time derivative of V_E along the solution of (9)-(11)-(13)-(15) is

$$\frac{d}{dt}V_E(t, e(t)) = -\sum_{i=1}^{4} e_i(t)^T K_i e_i(t),$$

thus

$$\frac{d}{dt}V_E(t,e(t)) \leq -\min\{\lambda_m(K_i), i=1,...,4\} \|e(t)\|^2 .$$

The global exponential stability of the origin for (9)-(11)-(13)-(15) follows from Theorem 1.2 [7].

Remark 3.2. It is straightforward to verify that if the origin is globally exponentially stable for system (9)-(11)-(13)-(15) then the origin is globally asymptotically stable for system (8) with (14).

4 A nonlinear state-observer

Consider system (8) with the output

$$y_1 = z_1 + q_d, \qquad y_2 = z_3 + p_d.$$

Let the state variables of the observer be denoted by ξ_i, $i = 1,...,4$, and let us estimate z_i in (8) by $\widehat{z}_i$, where

$$\widehat{z}_1 = \xi_1,\;\; \widehat{z}_2 = \xi_2 + k_1(z_1 - \widehat{z}_1),\;\; \widehat{z}_3 = \xi_3,\;\; \widehat{z}_4 = \xi_4 + k_2(z_3 - \widehat{z}_3), \tag{16}$$

where k_1 and k_2 are design parameters. The input variables of the observer are input and output variables of the original model (8) i.e. v, y_1 and y_2. The observer is looked for in the following form:

$$\begin{aligned}
&\dot{\xi}_1 = \xi_2 + k_1(y_1 - (\xi_1 + q_d)) + H_{11}(y_1 - (\xi_1 + q_d))\\
&B_1(y_1)\,\dot{\xi}_2 = -C_1(y_1, \xi_2 + k_1(y_1 - (\xi_1 + q_d)) + \dot{q}_d)(\xi_2 + k_1(y_1 - (\xi_1 + q_d)) + \dot{q}_d)\\
&\qquad -h_1(y_1) - K(y_1 - y_2) + \widetilde{H}_{21}(y_1 - (\xi_1 + q_d)) - B_1(y_1)\,\ddot{q}_d\\
&\dot{\xi}_3 = \xi_4 + k_2(y_2 - (\xi_3 + p_d)) + H_{32}(y_2 - (\xi_3 + p_d))\\
&\dot{\xi}_4 = B_3^{-1}K(y_1 - y_2) + B_3^{-1}v + \widetilde{H}_{42}(y_2 - (\xi_3 + p_d)) + B_3^{-1}u_d - \ddot{p}_d,
\end{aligned} \tag{17}$$

where H_{11}, $\widetilde{H}_{21}$,, H_{32}, $\widetilde{H}_{42}$ are matrices to be defined later. Using the notations

$$H_{21} = \widetilde{H}_{21} - k_1 B_1(y_1) H_{11}, \qquad H_{42} = \widetilde{H}_{42} - k_2 H_{32},$$

the behaviour of the variables $\widehat{z}_i$ is described by the equations

$$\begin{aligned}
&\dot{\widehat{z}}_1 = \widehat{z}_2 + H_{11}(y_1 - (\widehat{z}_1 + q_d))\\
&B_1(y_1)\,\dot{\widehat{z}}_2 = -C_1(y_1, \widehat{z}_2 + \dot{q}_d)(\widehat{z}_2 + \dot{q}_d) + k_1 B_1(y_1)(z_2 - \widehat{z}_2) - h_1(y_1)\\
&\qquad -K(y_1 - y_2) - B_1(y_1)\,\ddot{q}_d + H_{21}(y_1 - (\widehat{z}_1 + q_d))\\
&\dot{\widehat{z}}_3 = \widehat{z}_4 + H_{32}(y_2 - (\widehat{z}_3 + p_d))\\
&\dot{\widehat{z}}_4 = k_2(z_4 - \widehat{z}_4) + B_3^{-1}K(y_1 - y_2) + B_3^{-1}v + B_3^{-1}u_d - \ddot{p}_d\ .
\end{aligned} \tag{18}$$

By subtracting the equations of (18) from the corresponding equations of (8) we get for the observation errors

$$\widetilde{z}_i = z_i - \widehat{z}_i, \qquad i = 1,...,4$$

the following system of equations

$$\begin{aligned}
&\dot{\widetilde{z}}_1 = \widetilde{z}_2 - H_{11}\widetilde{z}_1\\
&B_1(y_1)\,\dot{\widetilde{z}}_2 = -[C_1(y_1, z_2 + \dot{q}_d) - C_1(y_1, \widehat{z}_2 + \dot{q}_d)]\widetilde{z}_2 - k_1 B_1(y_1)\widetilde{z}_2 -\\
&\qquad -H_{21}\widetilde{z}_1 - K(y_1 - y_2) - B_1(y_1)\,\ddot{q}_d + H_{21}(y_1 - (\widehat{z}_1 + q_d))\\
&\dot{\widetilde{z}}_3 = -H_{32}\widetilde{z}_3 + \widetilde{z}_4\\
&\dot{\widetilde{z}}_4 = -k_2\widetilde{z}_4 - H_{42}\widetilde{z}_3.
\end{aligned} \tag{19}$$

In the derivation of the second equation of (19) we have used property (5), to obtain the identity

$$\begin{aligned}
&C_1(y_1, z_2 + \dot{q}_d)(z_2 + \dot{q}_d) - C_1(y_1, \widehat{z}_2 + \dot{q}_d)(\widehat{z}_2 + \dot{q}_d) \pm C_1(y_1, z_2 + \dot{q}_d)(z_2 - \widehat{z}_2) =\\
&\quad = C_1(y_1, z_2 + \dot{q}_d)(\widehat{z}_2 + \dot{q}_d) + C_1(y_1, z_2 + \dot{q}_d)(z_2 - \widehat{z}_2) - C_1(y_1, \widehat{z}_2 + \dot{q}_d)(\widehat{z}_2 + \dot{q}_d) =\\
&\quad = C_1(y_1, \widehat{z}_2 + \dot{q}_d)(z_2 - \widehat{z}_2) + C_1(y_1, z_2 + \dot{q}_d)(z_2 - \widehat{z}_2).
\end{aligned}$$

Lemma 3.1. Let P_1, P_3, P_4, Q_1, Q_2 denote arbitrary symmetric and positive definite matrices. If the matrices H_{ij} are chosen according to the conditions

$$\begin{aligned} H_{11}^T P_1 + P_1 H_{11} &= Q_1, \quad H_{21} = P_1, \\ H_{32}^T P_3 + P_3 H_{32} &= Q_2, \quad H_{42} = P_4^{-1} P_3, \end{aligned} \tag{20}$$

then the time-derivative of the function

$$V_0(t, \widetilde{z}) = \frac{1}{2}[\widetilde{z}_1^T P_1 \widetilde{z}_1 + \widetilde{z}_2^T B_1(q_1(t))\widetilde{z}_2 + \widetilde{z}_3^T P_3 \widetilde{z}_3 + \widetilde{z}_4^T P_4 \widetilde{z}_4] \tag{21}$$

along the solution of (19) satisfies the relation

$$\frac{d}{dt} V_0(t, \widetilde{z}(t)) = -\frac{1}{2}\widetilde{z}_1^T Q_1 \widetilde{z}_1 - \widetilde{z}_2^T [C_1(y_1, \widehat{z}_2 + \dot{q}_d) + k_1 B_1(q_1(t))]\widetilde{z}_2 - \frac{1}{2}\widetilde{z}_3^T Q_3 \widetilde{z}_3 - k_2 \widetilde{z}_4^T P_4 \widetilde{z}_4. \tag{22}$$

Proof. By using property (5) and condition (20), (22) can be obtained by a simple computation.

5 Main result

In this section we investigate the effect of using $\widehat{z}_2$ and $\widehat{z}_4$ instead of z_2 and z_4, respectively, in (14). We shall show that the design parameters k_1 and k_2 can be chosen so that the corresponding dynamic output feedback semi-globally stabilizes system (8).

Let $Q \subset R^n$ be an arbitrary compact set containig the origin and assume that

$$\left\| q_d^{(i)}(t) \right\| \le M, \qquad \forall t \in [0, \infty), \qquad i = 0, ..., 4.$$

Let $S = B_M(0) \times B_M(0) \times B_M(0) \times B_M(0) \times B_M(0)$, where $B_\rho(z_0) = \{z \in R^{4n} : \|z - z_0\| \le \rho\}$. According to Remark 3.1, for $Q \times S$, there exists a constant $L > 0$ such that

$$\left\| \Phi_3(z_1, z_2, z_3, z_4, q_d(t), ..., q_d^{(IV)}(t)) - \Phi_3(z_1, \widehat{z}_2, z_3, \widehat{z}_4, q_d(t), ..., q_d^{(IV)}(t)) \right\| \le L(\|z_2 - \widehat{z}_2\|^2 + \|z_4 - \widehat{z}_4\|^2)^{1/2},$$

for any $(z_1^T, z_2^T, z_3^T, z_4^T)$, $(z_1^T, \widehat{z}_2^T, z_3^T, \widehat{z}_4^T) \in Q$ and for any $t \ge 0$.

Lemma 4.1. Assume that the conditions of Lemma 3.1 are satisfied, and the dynamic feedback (16) (17) and

$$\widehat{v}^*(t) = \Phi_3(z_1(t), \widehat{z}_2(t), z_3(t), \widehat{z}_4(t), q_d(t), ..., q_d^{(IV)}(t)) \tag{23}$$

is applied. If the trajectories $z(t)$, $\widehat{z}(t)$ remain in the compact domain Q during the whole motion, then k_1 and k_2 can be chosen so that the resulted in chosed-loop system is locally asymptotically stable.

Proof. Observe first that, using $\widehat{v}^*$ instead of v^*, gives error equations (9)-(11)-(13),

$$\dot{e}_4 = -K_4 e_4 - e_3 + B_3^{-1}(\widehat{v}^*(t) - v^*(t)) \tag{24}$$

and (19). Consider the Lyapunov function candidate

$$V(t, w) = V_0(t, \widetilde{z}) + V_E(t, e)$$

where $w^T = (e^T, \widetilde{z}^T)$. Calculate the time derivative of V along the solution of (9)-(11)-(13), (24) and (19). From (22) it follows that

$$\dot{V} = -\sum_{i=1}^{4} e_i^T K_i e_i - \frac{1}{2}\widetilde{z}_1^T Q_1 \widetilde{z}_1 - \frac{1}{2}\widetilde{z}_3^T Q_3 \widetilde{z}_3 - \widetilde{z}_2^T [C_1(q_1, \widehat{z}_2 + \dot{q}_d) + k_1 B(q_1)]\widetilde{z}_2 - \widetilde{z}_4^T k_2 P_2 \widetilde{z}_4 + e^4 B_3^{-1}(\widehat{v}^* - v^*).$$

The last term in this equation can be estimated as follows:

$$\begin{aligned} & e^4 B_3^{-1}(\widehat{v}^* - v^*) \le \tfrac{1}{2} e_4^T K_4 e_4 + 2(\widehat{v}^* - v^*)^T B_3^{-1} K_4^{-1} B_3^{-1}(\widehat{v}^* - v^*) \le \\ & \quad \le \tfrac{1}{2} e_4^T K_4 e_4 + 2\lambda_M((B_3 K_4 B_3)^{-1}) \|\widehat{v}^* - v^*\|^2 \le \\ & \quad \le \tfrac{1}{2} e_4^T K_4 e_4 + 2\lambda_M((B_3 K_4 B_3)^{-1}) L^2 (\|\widetilde{z}_2\|^2 + \|\widetilde{z}_4\|^2) \end{aligned}$$

Therefore,

$$\begin{array}{l}\dot{V} = -\sum_{i=1}^{3} e_i^T K_i e_i - \frac{1}{2} e_4^T K_4 e_4 - \frac{1}{2}\widetilde{z}_1^T Q_1 \widetilde{z}_1 - \frac{1}{2}\widetilde{z}_3^T Q_3 \widetilde{z}_3 - \\ -\widetilde{z}_2^T [C_1(q_1, \widehat{z}_2 + \dot{q}_d) + k_1 B(q_1) - 2\lambda_M((B_3K_4B_3)^{-1})L^2 I]\widetilde{z}_2 - \\ -\widetilde{z}_4^T [k_2 P_4 - 2\lambda_M((B_3K_4B_3)^{-1})L^2 I]\widetilde{z}_4. \end{array} \tag{25}$$

Because of the assumptions of the Lemma , there exist such positive constants α_1, α_2 that during the whole motion

$$\left\| z_2 + \dot{q}_d \right\| \leq \alpha_1, \qquad \|\widetilde{z}_2\| \leq \alpha_2, \tag{26}$$

thus from (4) it follows that

$$\left\| C_1(q_1, \widehat{z}_2 + \dot{q}_d) \right\| \leq C_{1M} \left(\left\| z_2 + \dot{q}_d \right\| + \|\widetilde{z}_2\| \right) \leq C_{1M}(\alpha_1 + \alpha_2)$$

Let β_2 and β_4 be arbitrarily fixed positive numbers, and let us choose the parameters k_1 and k_2 according to the inequalities

$$k_1 \geq \frac{1}{B_{1m}} [\beta_2 + C_{1M}(\alpha_1 + \alpha_2) + 2\lambda_M((B_3K_4B_3)^{-1})L^2] \tag{27}$$

$$k_2 P_4 \geq (2\lambda_M((B_3K_4B_3)^{-1})L^2 + \beta_4) I \tag{28}$$

Introduce the notations $\lambda_m(Q_i) = \beta_i$, $i = 1, 3$, $\lambda_m(K_i) = \kappa_i$, $i = 1, 2, 3$ and $\lambda_m(\frac{1}{2}K_4) = \kappa_4$, then

$$\dot{V}\ (t, w) \leq -\sum_{i=1}^{4} \kappa_i \|e_i\|^2 - \sum_{i=1}^{4} \beta_i \|\widetilde{z}_i\|^2 \tag{29}$$

Since

$$\gamma_1 \|w\|^2 \leq V(t, w) \leq \gamma_2 \|w\|^2, \tag{30}$$

where

$$\begin{aligned} \gamma_1 &= \frac{1}{2} \min\{1, B_{1m}, \lambda_m(P_1), \lambda_m(P_3), \lambda_m(P_4)\} \\ \gamma_2 &= \frac{1}{2} \max\{1, B_{1M}, \lambda_M(P_1), \lambda_M(P_3), \lambda_M(P_4)\}, \end{aligned}$$

the assertion of the Lemma follows from (29) and (30) by Theorem 1.2 [7]).

Lemma 4.2. Assume that for the solution of (9)-(11)-(13), (24) and satisfy (29) and (30). Then for any compact set of initial errors $E \times \widetilde{E} \subset R^{4n \times 4n}$ there exist a compact set $Q \subset R^{4n}$ such that $z(t) \in Q$, $(z_1(t)^T, \widehat{z}_2(t)^T, z_3(t)^T, \widehat{z}_4(t)^T)^T \in Q$.

Proof. From (29) and (30) it follows that

$$\|w(t)\| \leq \sqrt{\gamma_1/\gamma_2}\, \|w(0)\|$$

thus there exists a constant ρ_0 such that $\|w(t)\| \leq \rho_0$ for all $t \geq 0$. Since $z_1 = e_1$, we have that

$$\|z_1(t)\| \leq \rho_0, \qquad \forall t \geq 0.$$

From $e_2 = z_2 + K_1 z_1$ it follows that

$$\|z_2(t)\| \leq (1 + \|K_1\|)\rho_0 =: a_1 \rho_0$$

and

$$\|\widehat{z}_2(t)\| \leq \|z_2(t)\| + \|\widetilde{z}_2(t)\| \leq (1 + a_1)\rho_0$$

In the definition (10) of z_3^*, the function Φ_1 is continuously differentiable in its first two (vector) variables and $\Phi_1(0, 0, q_d, \dot{q}_d, \ddot{q}_d) = 0$, therefore there exists a constant $L_1 > 0$ such that

$$\left\| \Phi_1(z_1, z_2, q_d, \dot{q}_d, \ddot{q}_d) \right\| \leq L_1 \sqrt{\|z_1\|^2 + \|z_2\|^2}\ \text{, for all}\ z_1 \in B_{\rho_0},$$

Thus

$$\|z_3(t)\| \leq \|e_3(t)\| + \|z_3^*(t)\| \leq \rho_0 + L_1 \sqrt{1 + a_1^2}\rho_0 =: a_2 \rho_0.$$

Similarly to the considerations above by (12), one can state the existence of a constant $L_2 > 0$ such that

$$\left\|\Phi_2(z_1, z_2, z_3, q_d, \dot{q}_d, \ddot{q}_d, \dddot{q}_d)\right\| \le L_2\sqrt{\|z_1\|^2 + \|z_2\|^2 + \|z_3\|^2}\ , \text{ for any } z_1 \in B_{\rho_0},\ z_2 \in B_{a_1\rho_0},\ z_3 \in B_{a_2\rho_0}$$

therefore

$$\|z_4(t)\| \le \|e_4(t)\| + \|z_4^*(t)\| \le \rho_0 + L_2\sqrt{1 + a_1^2 + a_1^2}\rho_0 =: a_3\rho_0.$$

Moreover

$$\|\widehat{z}_4(t)\| \le \|z_4(t)\| + \|\widetilde{z}_4(t)\| \le (1 + a_3)\rho_0.$$

Thus the set Q can be taken as

$$Q = B_{\rho_0} \times B_{(1+a_1)\rho_0} \times B_{a_2\rho_0} \times B_{(1+a_3)\rho_0}$$

Remark 4.1. Observe, that by the above notations, α_1 and α_2 satisfying (26) can be chosen to be

$$\alpha_1 = M + a_1\rho_0, \qquad \alpha_2 = \rho_0$$

Theorem 4.1. Assume that the conditions of Lemma 3.1 are satisfied. Then for any given compact set $Z_0 \subset R^{4n}$ of initial conditions $z(0)$ and for any given compact set $\widetilde{Z}_0$ of initial observation errors $\widetilde{z}(0)$ of the form $\widetilde{Z}_0 = \{0\} \times \widetilde{K}_2 \times \{0\} \times \widetilde{K}_4$, the design parameters k_1, k_2 can be chosen so that the identically zero solution of (8) and (18) with the feedback (23) is semiglobally asymptotically stable with the domain of attraction $\mathcal{A} \subset R^{8n}$, where

$$\mathcal{A} = \left\{ \begin{pmatrix} z \\ \widehat{z} \end{pmatrix} \in: R^{8n} :\ z \in Z_0,\ z_1 = \widehat{z}_1,\ z_3 = \widehat{z}_3,\ z_2 - \widehat{z}_2 \in \widetilde{K}_2,\ z_4 - \widehat{z}_4 \in \widetilde{K}_4 \right\}$$

Proof. The proof immediately follows from Lemma 4.1 and Lemma 4.2 by observing that being $z_1(0) = y_1(0) - q_d(0)$ and $z_3(0) = y_2(0) - p_d(0)$ know values, $\widehat{z}_1(0)$ and $\widehat{z}_3(0)$ can be taken to be equal to them, i.e. $\widehat{z}_1(0) = z_1(0)$ and $\widehat{z}_3(0) = z_3(0)$.

6 Illustrative example

The performance of the proposed method has been tested by simulations with respect to a robot, the model and the system parameter values of which are given in [1] and [5].

Example 1. [5] The robot consists of one elastic joint, rotating in a vertical plane. Frictional forces have not been considered. Its dynamic model is represented by

$$\begin{aligned} J_L\ \ddot{q}_1 + k(q_1 - q_2) + \frac{1}{2}mgl\sin q_1 &= 0 \\ J_R\ \ddot{q}_2 - k(q_1 - q_2) &= u, \end{aligned}$$

where J_L and J_R are, respectively, the inertias of the link and of the motor rotor, m is the link mass, g is the gravity constant, l is the link length, and k is the elastic constant of the joint. The robot parameters are (all vallues are in SI units)

$$m = 1, \qquad l = 1, \qquad k = 100, \qquad J_R = 0.02, \qquad J_L = 0.4.$$

The desired reference trajectory for the link position is given by

$$q_d(t) = 1 - \frac{2}{1 + e^t},$$

thus

$$q_d(0) = 0, \qquad \dot{q}_d\ (0) = 0.5.$$

The initial conditions for the robot are

$$q_1(0) = 0.01, \qquad \dot{q}_1\ (0) = 0.5, \qquad z_1(0) = 0.01, \qquad z_2(0) = 0, \qquad z_3(0) = 0, \qquad z_4(0) = 0.$$

Figure 1:

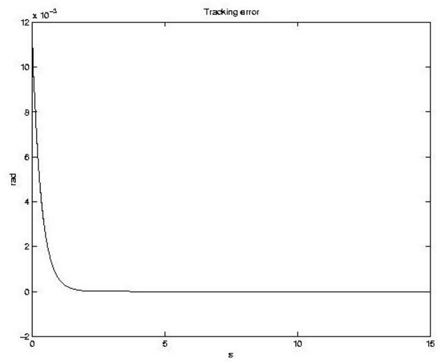
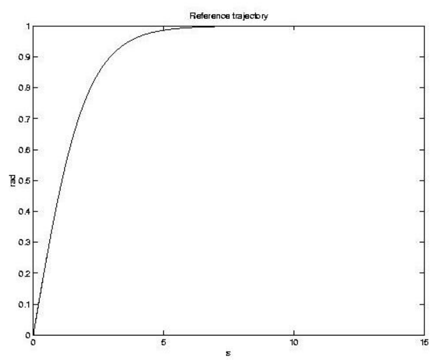


Figure 2:



and the initial conditions for the observer are

$$\widehat{z}_1(0) = 0.011, \quad \widehat{z}_2(0) = -5.6, \quad \widehat{z}_3(0) = 0, \quad \widehat{z}_4(0) = 0.$$

The design parameters are chosen to be

$$K_1 = 3, \quad K_2 = 9, \quad K_3 = 27, \quad K_4 = 225.$$

$$P_1 = 30, \quad P_3 = P_4 = H_{11} = H_{32} = 1, \quad H_{12} = H_{22} = H_{31} = H_{41} = H_{42} = 0,$$
$$H_{21} = 30.$$

The tracking error, the reference trajectory are reported in Fig 1, Fig 2.

Example 2. The robot $PUMA$ 560. The explicit dynamic model and inertial parameters have been published in [1]. The desired reference trajectory for the link position is given by

$$q_d(t) = [0, q_{d_2}(t), q_{d_3}(t), 0, 0, 0]^T,$$

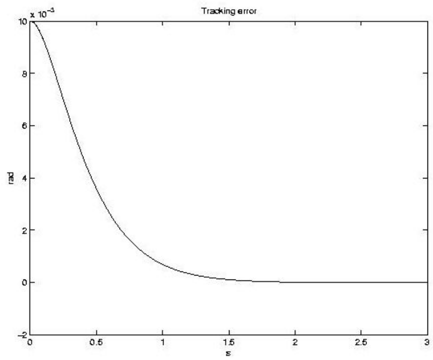
where

$$q_{d_2}(t) = q_{d_3}(t) = 1 - \frac{2}{1 + e^t}.$$

The initial conditions for the robot are

$$q_1(0) = [0, 0.01, 0.01, 0, 0, 0]^T, \quad \dot{q}_1\ (0) = [0, 0.5, 0.5, 0, 0, 0]^T.$$

Figure 3:



and the initial conditions for the observer are

$$\widehat{z}_1(0) = [0, 0.01, 0.01, 0, 0, 0]^T,\ \ \widehat{z}_2(0) = [0, 0, 0, 0, 0, 0]^T,$$

$$\widehat{z}_3(0) = [0, 0, 0, 0, 0, 0]^T,\ \ \widehat{z}_4(0) = [0, 0.01, 0.01, 0, 0, 0]^T.$$

The design parameters are chosen to be

$$L = 200, \qquad k_1 = 100, \qquad k_2 = 10.$$

$$P_3 = P_4 = H_{11} = H_{32} = I, \qquad H_{12} = H_{22} = H_{31} = H_{41} = 0, \qquad H_{22} = K,$$

$$P_1 = H_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 700 & 0 & 0 & 0 & 0 \\ 0 & 0 & 200 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$K_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 320 & 0 & 0 & 0 & 0 \\ 0 & 0 & 320 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \qquad K_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 980 & 0 & 0 & 0 & 0 \\ 0 & 0 & 980 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The tracking error and the reference trajectory are reported in Fig 3, Fig 4 for second link and in Fig 5, Fig 6 for third link.

Figure 4:

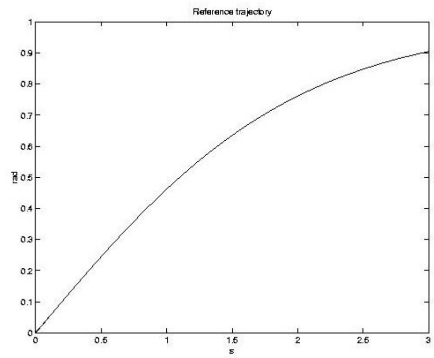


Figure 5:

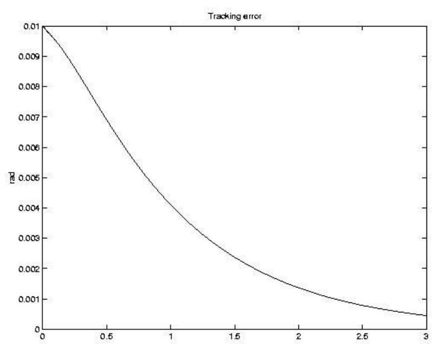
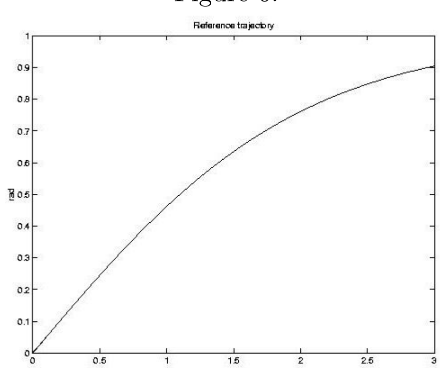


Figure 6:



7 Conclusion

The tracking control problem of flexible joint robots was considered under the assumption that the link and rotor positions are the only measured signals. A globally stabilizing state-feedback controller and a semiglobally asymptotically stable nonlinear observer was designed. Conditions for the observer design parameters were derived which ensured the validity of the separation principle for this special class of output feedback stabilization problem. Simulation examples illustrated the effectiveness of the proposed approach.

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