MANUFACTURING OF C-C PLANE GEARING WITH THE STANDARD AND MODIFIED RACK TYPE TOOLS

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Abstract

In spite of the plane convex-concave gearing (C-C gearing) is used in practice very rarely, in special cases it can solve the problems with load-capacity of wheels with involute type of flanks. The article deals with problem of non-involute C-C gearing production. It describes the production by such type of tools as are rack and hobbing cutter. There are also deals with possible modifications of the tooth flank profile of C-C gearing in terms of its manufacturing. The deviations of the tooth flank profile of wheel when the production was made with modified hobbing cutter are theoretically computed and graphically presented.

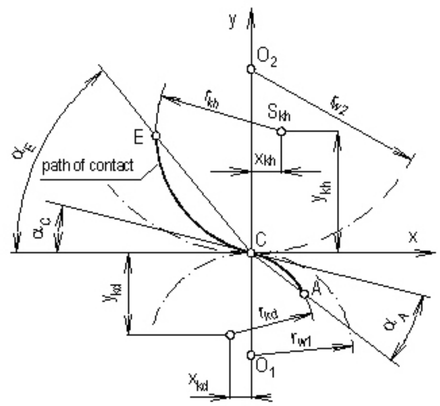

Figure 1: Common plane gearing the path of contact consisting of 2 circle arcs

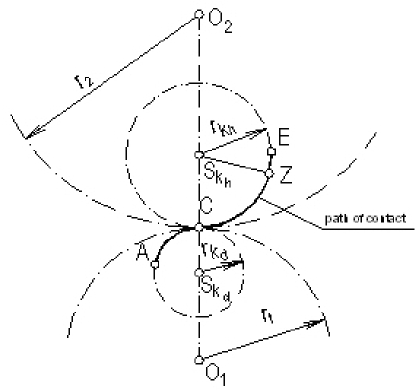

Figure 2: Path of contact of cycloidal gearing

1 Introduction

In conception of the common plane gearing and on the basis of [1] one can see a gearing that is defined by means of the path of contact consisted of 2 circlular arcs with their centres located on the arbitrary line passing through C point and the rotation centres of both mating gears, fig. 1.

At the same time the path of contact may have special forms as well, e.g. if it is identical with the centres of both toothed gears the matter in hand is a cycloidal gearing, fig.2, and moreover, if one arc radius of the generating line having the centre identical with the centre either of the toothed gears the matter in hand is a pin gearing, fig.3. In case that radii of both arc parts representing the path of contact possess infinite radii, circle arcs degenerate to a line which means that an involute gearing is defined. In common case the surface gearing definitions by means of the path of contact consisted of 2 circle arcs not having centres located on both toothed gear centres, have mating gear profiles mostly with convex-concave form (along with the convex-concave mesh). In world literature this kind of gearing is called as a convex-concave gearing, fig.4.

Generally, the path of contact may have a form of an arbitrary plane curve. If that curve is defined by the equation $r = r(\alpha)$, (fig.1), it is possible to solve the parametric equation forms of correctly meshing C-C pofiles by the process mentioned in [1]. According to this method we can obtain the parametric equations of curves of teeth flanks in form

$$x = \mp 2r_{kh,d} \sin(\alpha - \alpha_C) \cos(\alpha + \varphi_r(\alpha)) + r_1 \sin \varphi_r(\alpha)$$

$$y = \pm 2r_{kh,d} \sin(\alpha - \alpha_C) \sin(\alpha + \varphi_r(\alpha)) + r_1 \cos \varphi_r(\alpha) \qquad (1)$$

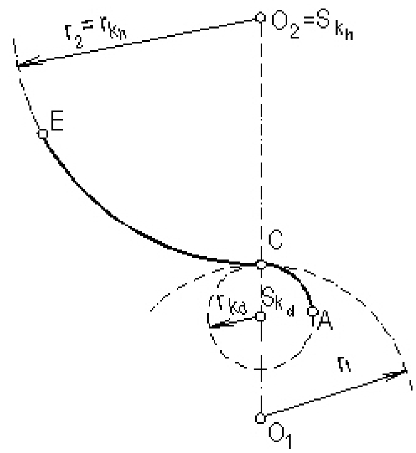

Figure 3: Path of contact of pin gearing, $r_k = r_2$

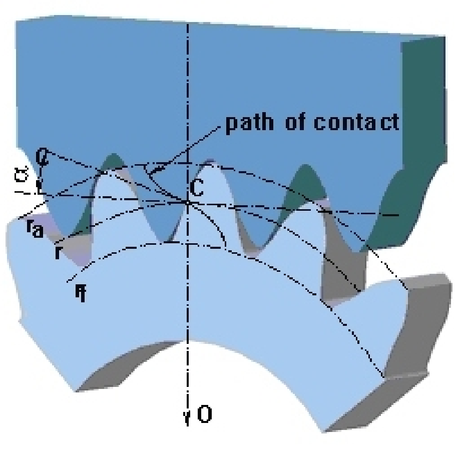

Figure 4: Convex-Concave Gearing

$$\varphi_r = \pm\frac{2r_{kh,d}}{r_1}[(\alpha-\alpha_C)\cos\alpha_C + \sin\alpha_C \lg\frac{\cos\alpha_C}{\cos\alpha}]$$

where φ_r is parameter which represents the wheel turn in a mesh point motion from the point C to the arbitrary point, which is determined through the pressure angle α.

Equations of the rack tool can be very easy determined as an extreme case of the equations (1) for $r_1 \longrightarrow \infty$. Then for a rack form flanks to the path of contact composed of two circular arcs we have

$$x^* = \mp 2r_{kh,d}\sin(\alpha-\alpha_C) \pm 2r_{kh,d}[(\alpha-\alpha_C)\cos\alpha_C + \lg\frac{\cos\alpha_C}{\cos\alpha}\sin\alpha_C]$$

$$y^* = \pm 2r_{kh,d}\sin(\alpha-\alpha_C)\sin\alpha \tag{2}$$

It is possible to show that equations of correctly meshing profiles of the spatial gearing, i.e. one having form defined by the path of contact in the shape of the spatial curve, can be derived by the same way. A special case of that spatial gearing is, e.g. the Wildhaber-Novikovov's gearing which is defined by the straight lined path of contact in the axial plane (perpendicular to the transverse plane).

A special favour of that general definition of the surface gearing acc. equations (1) are the following items:

1. The possibility of target option for gearing properties during their design by choice of the suitable form of path of contact.
2. The unified relations for all geometrical features of gearing irrespective of its kind (creation simplification of CAD/CAM systems with gearing design and continuous transition in features evaluation of variable types).
3. Simple features comparison of variable gearing sorts.
4. Uniform possibility of gearing model creation in case of the strength value calculation of FEM, etc.

According to the relation (1) it is admittedly possible to determine a type of gearing from the path of contact form but in connection with target option possibility of gearing properties during their design by choice of the suitable path of contact form it is necessary to be hard at work upon questions that are connected with geometric parameters of the particular gearing types especially of their correct meshing point of view along with choice of their basic geometric parameters. In world literature these questions are sufficiently elaborated theoretically for the cycloidal, pinted, Novikovov's and involute gearings. A case of convex-concave gearing is especially worked out in studies [1, 2, 3] and we will pay that theme main attention because many arguments of C-C gearing are possible to apply also to the other kinds of gearing. From the authors' research results in C-C gearing sphere of knowledge it has been followed that this kind of gearing displays a higher flexural loading, contact loading and scuffing (scoring) wear (up to 25%) in comparison with the involute gearing. In case of C-C gearing one of the main problems requiring a solution has been the question of its production.

The C-C gearing defined in [1] would be possible to produce by means of the rack-form cutters (Fig.5, Fig.6) by planing on the Maag's planer. According parametric equations (2) it is very easy to design and to manufacture the rack tool for the producing of concrete wheel with C-C gearing (i.e. with electroerosive machining or other NC and CNC machine tools). The virtual 3D model of such tool is shown on the Fig.5 and picture of real tool is displayed on the Fig.6.

On the other side the process of manufacturing the relevant wheel is not always appropriated. This manufacturing of wheels with C-C gearing, on the one hand has been a time consuming unreasonably expensive and at the same time on the other hand, considerably difficult from required engine stock standpoint. Therefore this kind of wheel machining is suitable only when high accuracy of gearing is required.

In the same way it is possible to manufacture the hobbing tools (fig.7) which is also a rack type tool. It is clear that the manufacture of such tool is more complicated. For that reason within the framework of grant projects research of VEGA 0296, 1/3184/06 and APVT 20-007602 authors have solved the question of its more efficient generation along with the possibilities of C-C gearing production by rack hobbingmill with the simple profile form where the compensatory profile of tool is consisted of 2 circle arcs. At the same time a tooth profile of gear wheel processed by that tool was observed. The examination method used mathematical method of hobbing.

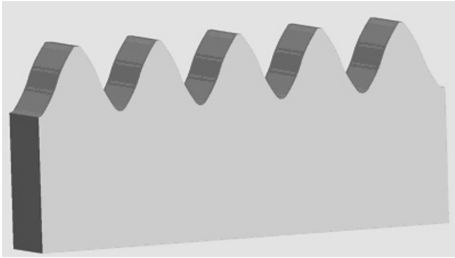

Figure 5: Model of Rack with Convex-Concave Shape of Flank Profiles

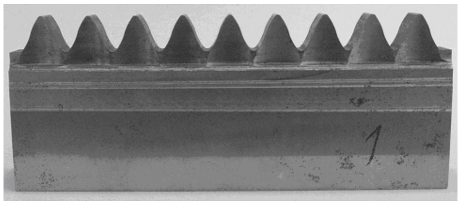

Figure 6: Real Rack Tool for machining of convex-concave gearing

Figure 7: Real hobbing cutter for machining of convex-concave gearing

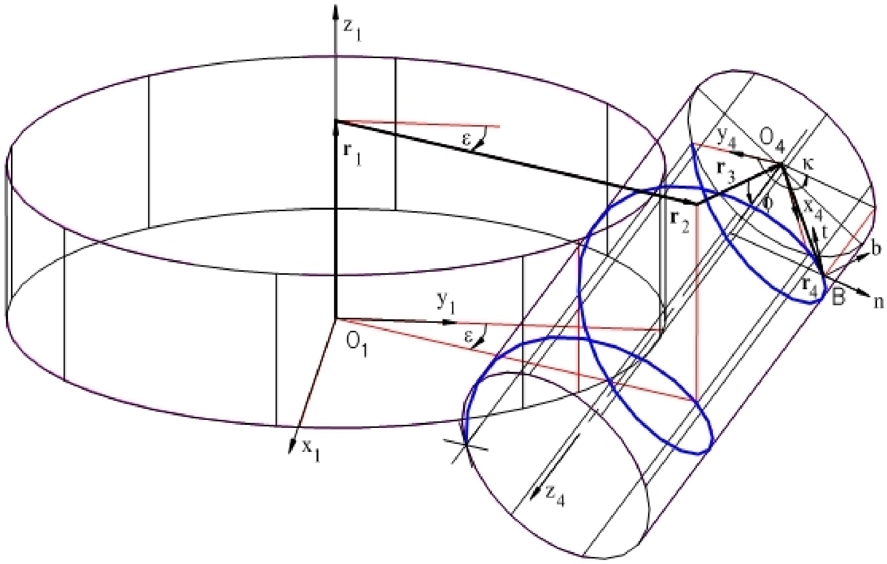

Figure 8: Creation model of tooth gap by hobbing process

2 Graphic-numerical model

Toothed gear is formed by the helical surface with profile set in the perpendicular (normal) plane. Forming surface moves in the direction of the tooth (with toothed gear having straight teeth in the direction of gear axis) and, at the same time the reference cylinder rolls over the reference cylinder of toothed gear. Model forming a gear tooth gap by hobbingmill is demonstrated in fig. 8.

Tool position vector $\mathbf{r}_n$ comprises vectors $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$, $\mathbf{r}_4$ and $\mathbf{r}_5$. Vector $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$ letters the tool starting point position-point one of O_4 on the tool axis

$$\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 = (R_1 + R_2 + k)\sin\varepsilon\mathbf{i}_1 + (R_1 + R_2 + k)\cos\varepsilon\mathbf{j}_1 + z_1\mathbf{k}_1 \tag{3}$$

Arbitrary point position on the tool helix B is indicated by vector $\mathbf{r}_4$ and tool profile form in the perpendicular plane (fig.9) is lettered by a position vector $\mathbf{r}_5$ of the N point.

$$\mathbf{r}_4 = -R_2\sin\kappa\mathbf{i}_4 + R_2\cos\kappa\mathbf{j}_4 + R_2\kappa\text{tg}\ \gamma\mathbf{k}_4 \tag{4}$$

Tool profile position vector in the perpendicular plane is indicated by the circle arcs

$$\mathbf{r}_{51} = (-R_{51}\sin\alpha_C + R_{51}\sin\delta_1)\mathbf{n} + (b_0 - R_{51}\cos\alpha_C + R_{51}\cos\delta_1)\mathbf{b} \tag{5}$$

$$\mathbf{r}_{52} = (R_{52}\sin\alpha_C - R_{52}\sin\delta_2)\mathbf{n} + (b_0 + R_{52}\cos\alpha_C - R_{52}\cos\delta_2)\mathbf{b} \tag{6}$$

Resulting tool position vector after modification is expressed by the following formula

$$\mathbf{r}_n = r_{xn}\mathbf{i}_1 + r_{yn}\mathbf{j}_1 + r_{zn}\mathbf{k}_1 \tag{7}$$

According to the envelope method [4] we get a searching surface if the A matrix determinants equal 0.

$$A = \left\| \begin{matrix} \frac{\partial\mathbf{r}_{xn}}{\partial z_1} & \frac{\partial\mathbf{r}_{xn}}{\partial\varepsilon} & \frac{\partial\mathbf{r}_{xn}}{\partial\kappa} & \frac{\partial\mathbf{r}_{xn}}{\partial\delta} \\ \frac{\partial\mathbf{r}_{yn}}{\partial z_1} & \frac{\partial\mathbf{r}_{yn}}{\partial\varepsilon} & \frac{\partial\mathbf{r}_{yn}}{\partial\kappa} & \frac{\partial\mathbf{r}_{yn}}{\partial\delta} \\ \frac{\partial\mathbf{r}_{zn}}{\partial z_1} & \frac{\partial\mathbf{r}_{zn}}{\partial\varepsilon} & \frac{\partial\mathbf{r}_{zn}}{\partial\kappa} & \frac{\partial\mathbf{r}_{zn}}{\partial\delta} \end{matrix} \right\| \tag{8}$$

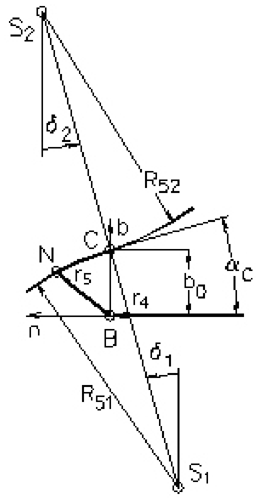

Figure 9: Tool profile shape in normal plane

Resulting tooth position vector will be getting by solution of (8)

$$\mathbf{r}_v = r_{xv}\mathbf{i}_1 + r_{yv}\mathbf{j}_1 + r_{zv}\mathbf{k}_1 \tag{9}$$

Deviation is evaluated as a difference between tooth profile in ideal set and tooth profile corresponding with required generating line in the perpendicular plane in the direction of binormal.

3 Practical hobbingmill proposal

For examination it was chosen the path of contact containing 2 symmetric arcs with inflection point in C one, with values of $r_k = 9$ mm and $\alpha_C = 11^\circ$ where we have solved one of the possible approximated profiles in the milling operation with hobbingmill. Toothed gear having the straight teeth with module $m_n = 4.575$ mm, $R_{51} = R_{52} = 13.5$ mm, angle profile $\alpha_C = 16^\circ$ is examined. Tool radius $R_2 = 37.5$ mm and tool setting angle $\Delta\gamma = 0^\circ$ were chosen. With tool set $\gamma = \varphi$, i.e. $\Delta\gamma = 0^\circ$ the tooth gap of gear corresponds precisely with the hobbed tooth profile.

4 Examination results

On the basis of answer to a problem of required milling cutter for production of C-C gearing mentioned above we subsequently compared the tool profile responding to required path of contact with approximated tool profile. Consequently, the gear teeth produced by hobbing were compared with both tools as well. Course of these deviations is demonstrated in fig. 10. According to the deviation course in fig. 10 we are able to deduce that change of tooth gap is not linear and causes also a profile curvature variation with this type of C-C tool profile. Nevertheless, after hobbing the tool deviation is approximately copied to the gear wheel. In the place of reference circle the deviation equals zero and tooth produced by approximated tool is thicker in required tooth. In spite of that there are so minimum dedendum area and is thinner in addendum one than values (of the order of hundredths of mm, e.i. less than 1% of tooth width) that stated results can be sufficiently considered as precise.

5 Conclusion

In the contribution one of the potential approximated C-C hobbed tooth profiles is examined. Gear wheel fitted with straight teeth is investigated. Mathematical model of considered production process with envelope method use is established. The tool profile responding to required path of contact is compared with approximated tool profile.

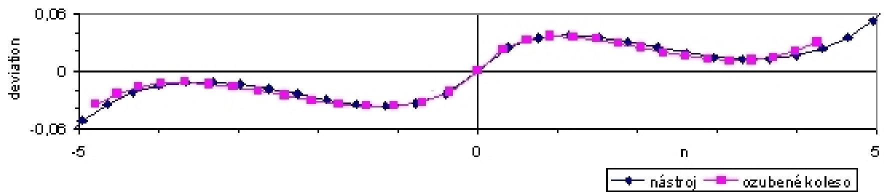

Figure 10: Deviation courses of hob and wheel

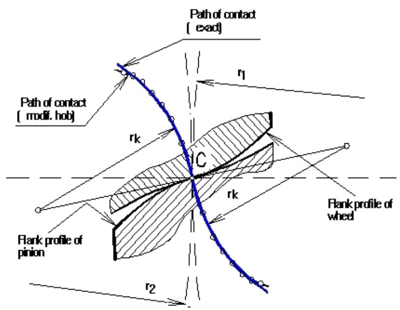

Figure 11: Path of contact of mating gears produced by theoretically correct and modified tool

Moreover, the gear teeth produced by hobbing are compared with these tools. We can observe, that tooth profile of wheel produced with modified tool differs from the theoretically exact one insignificantly. According to analysis of impact of tool profile modification on path of contact shape it is also evident (fig.8), that deviation occurs only at the end of the path of contact. Documented examination reveals the theoretical investigation possibilities of various profiles along with their optimum improvements. By utilization of detected dependance it is possible to reach required profile with satisfactory accuracy. By the process described in this article it is also possible to inquire an optimal way of production and profile form but a mathematical model of considered production process is necessary to establish.

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