# CW and FM-CW radar adaptation for vehicle technology

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#### Abstract

This series of articles deals with the theory and characteristics of near and far-reaching, continuous and impulse driven radars which implement safety measures in vehicles. In the first article we discuss the theory of CW and FM-CW radars.

# **1** Introduction

Related to the convergence theory, radar technology more and more cover the whole area of transportation, even more its role is increasingly growing in vehicle technology, significantly raising its safety by the aid of speed, distance and distance resolution measurements.

There are many types of radar used in vehicles, but this article only covers radars in continuous operation. The theory explained here is aided by the design and construction of radars which are built into vehicles. The main reason for writing this article is the need for knowledge about the theory and practice of radar type theory. There are three main principles in this area: the constant frequency reflected signals by Doppler shift measurement, the reflected signals phase shift (Interferometry) and the reflected FW-CW signals parameter measurement. Henceforward, let's take the functional principle of each type one by one.

# 2 CW Doppler radar

## 2.1 Doppler theory explanation

According to the Doppler theory: if there is a continuous signal (CW) transmitting transmitter, it beams an  $f_a$  frequency on a target (for example a vehicle) and the target has a relative speed compared to the transmitter  $(v_r)$ , then the reflected signal frequency  $(f_v)$  is larger or smaller than  $f_a$  depending on whether the target is diverging or coming closer relative to the transmitter.[1][2] The covered principle is shown in Figure 1. Figure 1a shows a steady wave source, Figure 1b. shows a right-moving source and Figure 1c shows a left-moving source.

Figure 1: Wave images of stationary and moving sources



The received signal which is different from the transmitted signal is named Doppler Frequency  $(f_D)$ . For numeric analysis, let's take a look at the Doppler Frequency in Figure 2, where you can see one CW transmitter emitted electromagnetic wave for one period. The  $\overline{AB}$  wave section moves with the speed of light, (c is the speed of light). The end of wave section B reaches A within t time. HEJ: TAR-070416-A

This time is:

$$t = \frac{d}{c}.$$



If there is a target (vehicle) in the way of the wave section, it is lighted for the same amount of time. Let's move the vehicle in the direction of the transmitter (or in the direction of the other vehicle with built-in radar which standing or moving)  $v_r$  radial speed. In that case, the radar wave speed compare the target is  $c + v_r$ . Same time is necessary to take the d distance mark with  $t_1$  and its value is:

$$t_1 = \frac{d}{c + v_r}.\tag{2}$$

Meanwhile the target covers a distance:

$$s = v_r \cdot t_1 = v_r \frac{d}{c + v_r}.\tag{3}$$

The B end of the wave has to travel that amount of less distance from the target to the radar transmitter. The time necessary to take the distance  $(t_2)$ :

$$t_2 = \frac{s}{c} = \frac{v_r \cdot t_1}{c} = \frac{v_r \cdot d}{c(c+v_r)}$$

As shown in Figure 2. the wavelength is "shorter" which is expressed in time:

$$t_1 - t_2 = \frac{d}{c + v_r} - \frac{v_r \cdot d}{c(c + v_r)} = \frac{d}{c} \cdot \frac{c - v_r}{c + v_r}.$$
(4)

From the 4th expression excel, that the reception time compare the transmission time  $(c - v_r)$ ,  $(c + v_r)$  rate decreased, but the decreased time the same period is arrive back, compare the amount emitted from the transmitted, which means that the frequency is increased meanwhile.

The amount of increase is in relation with the reduced time, therefore the received frequency for one period  $(d = \lambda)$ :

$$f_v = \frac{1}{t_1 - t_2} = f_a \cdot \frac{c + v_r}{c - v_r}.$$

The difference in frequency is the Doppler - frequency:

$$f_D = f_v - f_a = \frac{2 \cdot v_r}{c - v_r} f_a.$$
(5)

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(1)



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Taking into account that the target speed is irrelevant compared the light speed:

$$f_D = 2\frac{v_r}{c}f_a = 2\frac{v_r}{\lambda} \Longrightarrow v_r = \frac{f_D \cdot \lambda}{2}.$$
 (6)

From the previous, these can also be written:

$$\omega_D = 2\pi \cdot f_D = \frac{d\Theta}{dt} = \frac{2\pi}{\lambda} \cdot \frac{dr}{dt} = \frac{4\pi \cdot v_r}{\lambda}.$$
(7)

Where  $\Theta$  marks the momentary phase. Additionally the following expression can be written:

$$f_D = \frac{2v_r}{\lambda} \approx 55, 5 \frac{v_r[km/h]}{\lambda[cm]} [Hz] \approx 1,85 \cdot f_t[GHz] \cdot v_r[km/h] [Hz].$$

$$\tag{8}$$

Figure 3 shows the block scheme of a simple CW radar.[3]

Figure 3: Block scheme of a simple CW radar



In real cases, there are two antennas but for simplicity only one is shown here. The continual (radiated) and the reflected signal difference is the Doppler frequency. Low frequency is hard to process, therefore a critical situation can come into existence, therefore the minimum measured vehicle speed is determined, as well as the practical base frequency and the esti-mated amount of noise and sensitivity.

The vehicles radar receiver like any other receiver is based on the superheterodin principle. (Figure 4.).

Figure 4: The vehicles radar receiver



Figure 5 shows the CW radar signal typical layout in the frequency range.[4]

The built-in car radars frequency contain the quadrature modulation. Let  $i_1(t)$  and  $q_1(t)$  be two real numbers, proportional to the  $\omega_h$  frequency bandwidth limited signal, whose spectrum is  $I_1(\omega)$  and  $Q_1(\omega)$ . (A signal is band limited, if the following conditions are met:  $I_1(\omega) = Q_1(\omega) = 0$ , ha  $|\omega| > \omega_h$ .)

Note: The i signal is called in-phase, the q signal is called quadrature-phase signal.

The transmitter emitted signal:

$$u(t) = i_1(t)\cos\omega_v t - q_1(t)\sin\omega_v.$$
(9)

Where the carrier frequency:  $\omega_v >> \omega_h$ . The frequency range:

### Figure 5: Typical layout of CW radar



$$U(\omega) = \frac{1}{2} [I_1(\omega - \omega_v) + I_1(\omega + \omega_v)] + j\frac{1}{2} [Q_1(\omega - \omega_v) - Q_1(\omega + \omega_v)] =$$
  
=  $\frac{1}{2} [I_1(\omega - \omega_v) + jQ_1(\omega - \omega_v)] + \frac{1}{2} [I_1(\omega + \omega_v) - jQ_1(\omega + \omega_v)].$  (10)

In general the emitted radar signal is:

$$u(t) = a(t) \cdot \cos[\omega_v t + \varphi(t)]$$

This unfolded look like this:

$$u(t) = u_I(t)\cos(\omega_v \cdot t) - u_Q(t)\sin(\omega_v \cdot t)$$

Where the phase component is:

$$U_I(t) = a(t) \cos[\varphi(t)].$$

And the quadrature component is:

$$U_Q(t) = a(t)\sin[\varphi(t)]$$

therefore:

$$U_0(t) = U_I(t) + jU_Q(t) = a(t) \cdot \exp[j\varphi(t)].$$
(11)

As a vector mark the real part with I, and the imaginary part with Q.

Figure 6: Both marks for the different values



Further relationships based on the Figure 6 are:

$$U_0 = \sqrt{I^2 + Q^2}; \quad \varphi = \operatorname{arctg} \frac{Q}{I}; \quad f = \frac{1}{2\pi} \cdot \frac{d\varphi}{dt}.$$

Let's analyze the vehicle's CW radar modulator and demodulators (Figure 7.)[6]



Figure 7: Vehicle's CW radar modulator and demodulators

Figure 8: The signals frequency range



The signals frequency range image is shown in Figure 8. The detector is first mixed with a coherent, two phase oscillator signal base band:

$$i_2(t) = U(t)\cos\omega_v t = \frac{1}{2}[i_1(t) + i_1(t)\cos 2\omega_v t - q_1(t)\sin 2\omega_v t].$$
(12)

$$q_2(t) = -U(t)\sin\omega_v t = \frac{1}{2}[q_1(t) - q_1(t)\cos 2\omega_v t - i_1(t)\sin 2\omega_v t].$$
(13)

In the second step, both channel frequencies are filtered  $2\omega_v t$  by low-pass filter and then when the low-pass filter is ideal, the result will be:

$$I = i_3(t) = \frac{1}{2}i_1(t) \quad \text{és} \quad Q = q_3(t) = \frac{1}{2}q_1(t).$$
(14)

Which means that the two channel signals can successfully be restored on the receiver side. The same result can be achieved by different methodologies applied in vehicle radars, too. In that case, we conduct the common oscillator signal into the two mixers with a 90° phase shift, and the output contains the I and Q values. That method is simpler (Figure 9.).

The CW radar in general is used to measure speed, which is based on the Doppler shift. The simple CW radar won't be able to measure the Doppler shift sign - the changed direction - nor the target distance, however with special integrated circuits this is possible.

#### Figure 9: Quadrature modulation



# 3 FM - CW radar

#### **3.1** Distance measurement

In case we plan to use the CW radar for distance measurement, the transmitter frequency has to be modulated.[3] The prevalent method is that transmitter frequency is changed over time. The difference correlated to the middle frequency of the transmitter oscillator (f0) is increased or decreased by some function. This function can be triangular, sawtooth or sinusoidal in shape. In radars, in general, the first two are in use.[8]

In these cases we could talk about linear frequency modulation. This method uses a frequency for the signal which changes linearly over time. This frequency is called chirp. One whole contains 4 chirps. The goal of linear frequency modulation is to establish a simple system for distance measurements. In case of linear frequency modulation, there is difference in frequency only between the radiated and reflected signal (df). If the target is standing still, this difference is distance-harmonic (R).

Examine the transmitted and the received signal momentary frequency, if  $v_r = 0$ . The examination is based on Figure 10.



Then the following expression is valid:

$$df = -2\frac{f_a}{c \cdot T}R.$$
(15)

In practice the reflected signal has two frequency differences and their sum gives the whole frequency difference (df). One component is the runtime frequency difference  $(f_{\tau} - \text{related to the distance})$ , the other component  $f_D$  came from the Doppler shift frequency difference  $(f_D, \text{related to the speed})$ .

This ensures that the target distance and speed can have multiple values.

Examine Figure 10b:

The different frequencies are:



$$f_D = -2\frac{v_r}{\lambda}$$

$$f_\tau = -2\frac{f_a}{c \cdot T}R$$

$$df = f_D + f_\tau,$$
(16)

In the simple measurement of the frequency it is not possible to separate the difference related to the distance and related to the Doppler shift.

That indefiniteness is shown in the Figure 11b. (Doppler distance) diagram. Examine Figure 11a., which is based on this expression:

$$df_1 = f_D + \left(-2\frac{f_{a1}}{c \cdot T}\right)R\tag{17}$$

and this expression:

$$f_D = -2\frac{v_r}{\lambda}$$

The classical solution is where the measurement indefiniteness can be resolved with the help of and other chirp. That leads to a two-variables equation, when resolved we get a value-pair for both distance and relative speed. With linear frequency modulation the scan rate (df/dt) and the scan bandwidth  $(f_a = f_1 - f_0)$  correlate as  $df/dt = (f_a/T_{chirp})$ . The Near Distance Sensor waveforms we utilities chirps, which time period is  $T_{cirp}$ . The first chirp case uses the  $f_0 - f_1$  frequency range. However, the frequency increases continuously, therefore the second chirp takes over the first chirp's place, which is called an upchirp. At the end the frequency returns to  $f_0$ . That is called a downchirp. The two measurements lead to the following two equations:

Figure 11a shows the  $R \dots f_D$  characteristics. Knowing the df1 and  $df_2$  we get two lines, whose intersection shows the correct solution (Figure 11b.).

In Figure 12, unified signal schemes are shown on which the later shown block schemes can be built.

In Figure 12.,  $f_a(t)$  means the modulated radiated signal,  $f_v(t)$  means the received signal  $[f_v(t) = f_a(t - \tau) \pm f_D(t)]$ ,  $f_A$  is the difference signal and  $f_m$  is the modulation frequency. In Figure 12. the continuous line is a transmitted signal and the dotted line shows t = 2R/c the received signal parameters with time-delay. The  $\Delta f$  is defined by the amplitude of the signal for modulation inside the transmitting oscillator driver bandwidth, the maximum measurement distance is  $f_m$  value. The  $f_T$  is the frequency related T time-period, the  $f_R$  is the frequency related to the target distance.[3][5]

The transmitted frequency:

- In the case of the sawtooth modulation signal:

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Figure 12a.

$$f_T = f_0 + \Delta f \frac{t}{T}.$$
(19)

- In the case of the triangular modulation signal:

$$f_T = f_0 + \Delta f \frac{2t}{T}.$$
(20)

- In the case of the sinusoidal modulation signal:

$$f_T = f_0 + \Delta f \frac{t + \Delta t}{T}.$$
(21)

While the electromagnetic waves travel the distance between the vehicle and the transmitter, the transmitter frequency changes and the difference between the two frequencies is mixed by the mixer to its output. This frequency is called beat frequency  $(f_B)$ . The value for the beat frequency is

$$f_B = \left(f_0 + \Delta f \frac{t + \Delta t}{T}\right) - \left(f_0 + \Delta f \frac{t}{T}\right).$$
(22)

The distance is:



$$R = c \frac{f_B}{2\Delta f} T.$$
(23)

In case of triangular modulation the Figure shows that in target in R distance the frequency is in case of  $t = 2R/c = 1/4f_m$  time-delay result  $f_B = \Delta f/2$  frequency difference.[4] From that condition the beat frequency is:

$$f_B = \frac{4f_m \cdot \Delta f \cdot R}{c}.$$
(24)

Therefore:

$$R = \frac{c \cdot f_B}{4f_m \cdot \Delta f}.$$
(25)

When  $f_m$  is given the maximal range is get when  $t = \frac{1}{2}f_m$ , because then  $f_B = \Delta f$  and the distance related are:

$$R_M = \frac{c}{4f_m}.$$
(26)





#### 3.2 Range resolution measurement

The vehicle's safe movement not only depends on the speed and distance of the vehicles in from the us, but the resolution (the ability to separate two vehicles from each other) measurement is required. In case when there are two targets, the reflected signals are received in different time. In that case:

$$\Delta f_B = |f_{B1} - f_{B2}| = \frac{2\Delta f}{c \cdot T} \Delta R.$$
(27)

Where:  $f_{B1}$  and  $f_{B2}$  the  $R_1$  and  $R_2$  distance related beat frequency. Moreover:

$$\Delta f_{B_{min}} = \frac{1}{T} = \frac{2\Delta f}{c \cdot T} \Delta R_{min}.$$

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From this:

$$\Delta R_{min} = \frac{c}{2\Delta f} \tag{28}$$

This is inversely proportional to the bandwidth. Apparently, the decrease of the measurement period decreases the resolving capacity. In practice this could lead up to a 30% decrease. Shown as:

$$\Delta R_{min} = 1, 3\frac{c}{2\Delta f}$$

Meanwhile  $R_{max}$  determines the time interval of the period. The correct operation requires the following condition:  $T >> 2R_{max}/c$ .



Figure 13. FM-CW radar which uses triangular modulation for operation and  $I/{\rm Q}$  demodulation

The other types of vehicle radars and their signals and signal processing methods related to radars introduced in this article will be the subject of the next article.

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